MA-161 (F,07)
Information & Formulas
For Test 2

GENERAL INSTRUCTIONS: Do all the problems on the paper I've provided unless otherwise directed. (Problem 5 is to be done on the special page. Only Problem 7 is to be done on the test paper.) Be sure you number the pages and the problems clearly. Answer all problems in sentences, even those requiring a numerical or formulaic answer. (For example, if you were asked to differentiate $y = x^2$, your answer would be the sentence $y' = 2x$.) Be sure to show your work. Your work, as well as your answers, will be graded.

A number of formulas are on the Precalculus Formula Sheet, including the Factorization Formula and some exponent and log laws. Also, if you need a formula or can't remember how to do something, ask me and I'll (possibly) tell you.

Here are some of the derivative rules we derived: Assuming u is a function of x,

$\frac{d}{dx}(u^n) = n \cdot u^{n-1}\frac{du}{dx}$ $\qquad \frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$ $\qquad \frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$ $\qquad \frac{d}{dx}(\tan u) = \sec^2 u\frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$ $\qquad \frac{d}{dx}(\ln u) = \frac{1}{u}\cdot\frac{du}{dx}$ $\qquad \frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\qquad \frac{d}{dx}(\arctan u) = \frac{1}{1+u^2}\frac{du}{dx}$ (See below.)

PLEASE: DO NOT FORGET TO USE THE CHAIN RULE, THE PRODUCT RULE, THE QUOTIENT RULE, ETC. WHEN THEY APPLY!

Also, you may use these special limits where necessary:

$\lim_{h\to 0}\frac{e^h - 1}{h} = 1$ $\qquad \lim_{h\to 0}\frac{\sin h}{h} = 1$ $\qquad \lim_{h\to 0}\frac{\cos h - 1}{h} = 0$ $\qquad$ If P is a polynomial, then $\lim_{x\to a}P(x) = P(a)$.

Here is the derivation of the formula for $\frac{d}{dx}(\arctan x)$, using the derivative of an inverse function procedure and implicit differentiation:

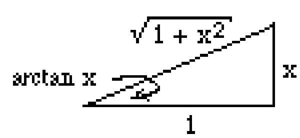
Let $y = \arctan x$. Then $\tan y = x$. Differentiate both sides with respect to x to get

$\frac{d}{dx}(\tan y) = \frac{d}{dx}(x)$. Since y is a function of x, the left side is $\sec^2 y\cdot\frac{dy}{dx}$ and the right side is 1.

$\therefore \sec^2 y\cdot\frac{dy}{dx} = 1$. Solve this equation for $\frac{dy}{dx}$:

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$. Since $\frac{1}{\sec y} = \cos y$ and $y = \arctan x$, then $\frac{dy}{dx} = (\cos y)^2 = (\cos \arctan x)^2$.

From trigonometry, we know $\cos \arctan x = \frac{1}{\sqrt{1+x^2}}$ so



$$\frac{dy}{dx} = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 = \frac{1}{1+x^2}.$$

$$\therefore \frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}.$$